

## Students Enrolled in Calculus I or II 2020-2021

### Summer Packet

Name: \_\_\_\_\_

This packet is to help you review various topics that are considered to be prerequisite knowledge upon entering Calculus. In order to ensure that the good skills that you have developed in math do not disappear this summer, working on this packet is highly recommended over the summer. (A good habit would be to do at least one math problem every day.) Enjoy your summer, but be sure to come prepared with the necessary knowledge to continue on into Calculus next year. There will be a skills quiz on these topics in the fall.

### I. Algebra

Factor the following

$$x^2 - 2x - 8$$

$$4x^2 + 5x - 6$$

$$2x^2 + xy - 15y^2$$

$$x^2 - 4$$

$$a^4 - b^4$$

$$x^2 - 4x + 4 - y^2$$

Perform the indicated operations and simplify

$$\frac{x^2 - x - 6}{x^2 - 6x + 9}$$

$$\frac{x^2 + 4x + 4}{x^2 - 3x - 4} \cdot \frac{2x^2 + x - 1}{x^2 - 3x - 10}$$

$$\frac{2x^2 - 6x}{2x^2 - x - 10} \div \frac{x^2 - 5x + 6}{x^2 - 4}$$

$$\frac{1}{x-1} + \frac{3}{x+2}$$

$$\frac{x}{2x-1} - \frac{3}{x+2}$$

$$\frac{\frac{1}{a^2} - \frac{2}{ab}}{\frac{1}{2ab} - \frac{1}{b^2}}$$

Use any method to solve the equation:

$$6x = 3x^2$$

$$x^2 - 4x - 5 = 0$$

$$4x^3 - 6x^2 = 0$$

$$\sqrt{x+4} = 3$$

$$|2x+3| = 7$$

$$\frac{1}{2x} + \frac{5x}{8} = 6$$

$$2x^2 - 8x = -3$$

$$(t+1)(2t-1) = 1$$

$$12 - 2s = 3s^2$$

Find the equation of the line (in point-slope form) that passes through each set of points:

$$(2, 1), (14, 6)$$

$$(-1, 0), (6, 2)$$

$$(1, 6), (4, 2)$$

Determine whether the following functions are odd, even, or neither:

$$x^2 + 2$$

$$2x^3 - x$$

$$x^4 - 3x$$

Determine the domain and range of each function:

$$(x - 1)(x + 2)$$

$$3 - 2x^2$$

$$\sqrt{2x^2 - 1}$$

$$\frac{2}{3x + 4}$$

$$|x + 5|$$

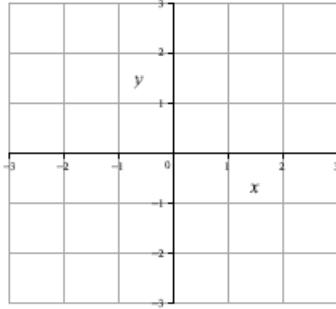
$$\sqrt{36 - x^2}$$

$$\sqrt{3 - x - 2x^2}$$

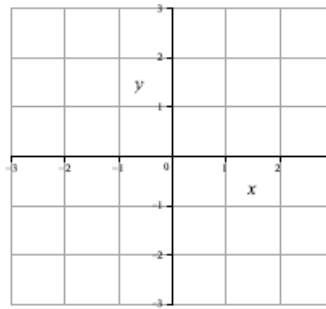
$$\sqrt{\frac{x + 3}{x^3 - 4x^2}}$$

$$\frac{1}{\sin x}$$

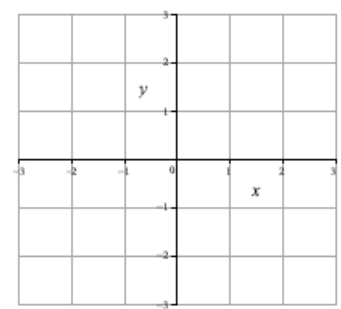
Sketch the graph of each function and determine its domain:



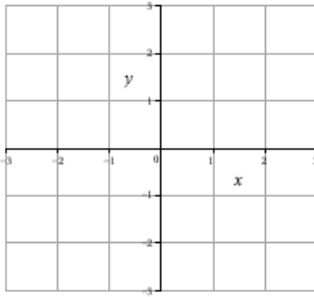
$$f(x) = x + 1$$



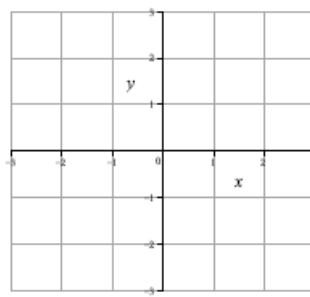
$$f(x) = 2x - 3$$



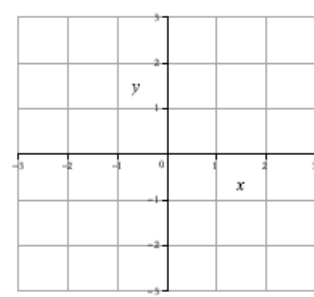
$$f(x) = \frac{1}{x - 1}$$



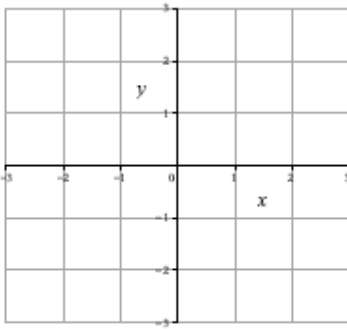
$$f(x) = \sqrt{2 - x}$$



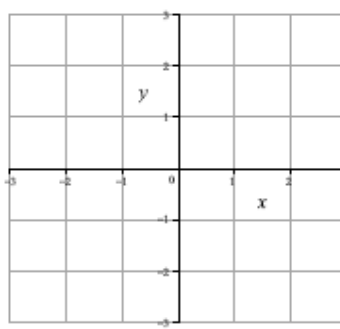
$$f(x) = \sqrt{9 - x^2}$$



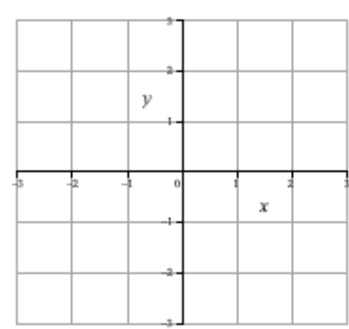
$$f(x) =$$



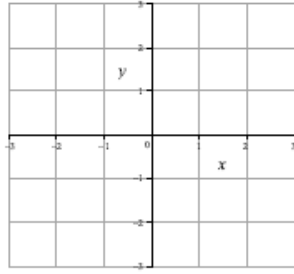
$$f(x) = |x - 1|$$



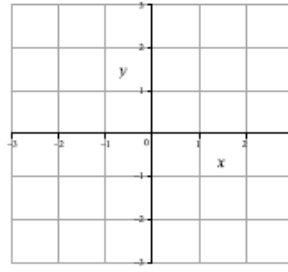
$$f(x) = \frac{x^2 - 1}{x + 1}$$



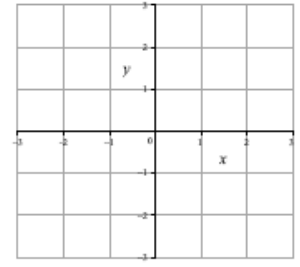
$$f(x) = x^2 - 6$$



$$f(x) = \begin{cases} 1, & x < 0 \\ x + 1, & x \geq 0 \end{cases}$$



$$f(x) = \begin{cases} x^2 - 1, & x < 1 \\ x, & x > 1 \end{cases}$$



$$f(x) = \begin{cases} x + 1, & x < -1 \\ 1 - x, & -1 \leq x < 1 \\ x, & x \geq 1 \end{cases}$$

**Composition of Functions** If  $f(x) = x + 2$  and  $g(x) = \sqrt{x}$ , find each function and its domain:

$$(f + g)(x)$$

$$(f - g)(x)$$

$$(fg)(x)$$

$$\left(\frac{f}{g}\right)(x)$$

$$f(g(x))$$

$$(f \circ g)(x)$$

$$g(f(x))$$

$$f(f(x))$$

**Given the function  $f(x) = x^3 - 2x^2 + 2x - 4$ , answer the following questions:**

- (a) What is the degree of this function?
- (b) How many complex roots does it have?
- (c) What is the minimum number of real roots that can you be sure that it has?
- (d) What is the domain in which all of its real roots must lie?
- (e) What are its potential rational roots?
- (f) Determine its real root(s) using your calculator

**Given the function  $f(x) = x^4 + x^3 - 2x^2 - 7x - 4$ , answer the following questions:**

- (a) What is the degree of this function?
- (b) How many complex roots does it have?
- (c) What is the minimum number of real roots that can you be sure that it has?
- (d) What is the domain in which all of its real roots must lie?
- (e) What are its potential rational roots?
- (f) Determine its real root(s) using your calculator



Complete the following:

$$\cos^2 x + \sin^2 x = \underline{\hspace{2cm}}$$

$$1 + \tan^2 x = \underline{\hspace{2cm}}$$

$$1 + \cot^2 x = \underline{\hspace{2cm}}$$

$$\sin 2x = \underline{\hspace{2cm}}$$

$$\tan\left(\frac{\pi}{2} - x\right) = \underline{\hspace{2cm}}$$

$$\cos(-x) = \underline{\hspace{2cm}}$$

Find all solutions for each of the following.

$$\tan x - 3 \cot x = 0$$

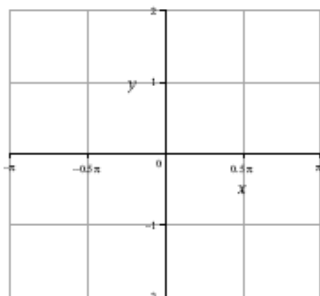
$$\sqrt{3} \cos x = \sin 2x$$

$$2 \cos^2 x - \cos x - 1 = 0$$

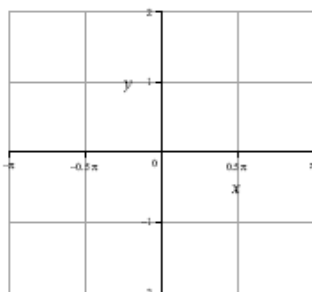
$$4 \cos^2(2x) = 1$$



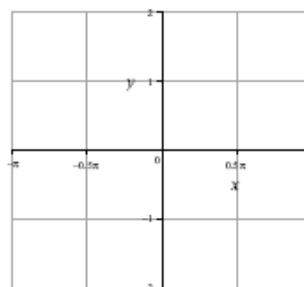
Sketch the graph of each function and state its domain and range:



$$f(x) = \sin x$$



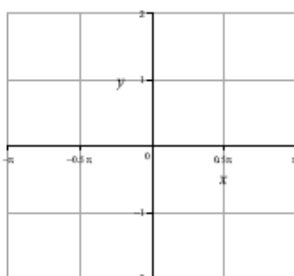
$$f(x) = \cos x$$



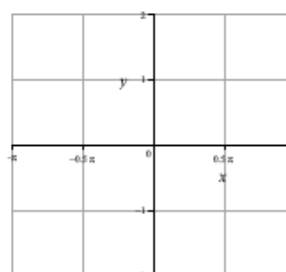
$$f(x) = \tan x$$



$$f(x) = \csc x$$



$$f(x) = \sec x$$



$$f(x) = \cot x$$

## Logarithms

Use the properties of logarithms and exponents to rewrite each of the following.

$$2^{x+2}$$

$$\log_6 \frac{5}{x}$$

$$\log_{10} x^2 y^4$$

$$2 \ln 8 + 5 \ln z$$

$$\ln \frac{xy}{z}$$

$$2 \ln 3 - \frac{1}{2} \ln(x^2 + 1)$$

$$\log_{10} \frac{x^4 \sqrt{x}}{z^4}$$

$$\ln e^2$$

$$e^{\ln x^2}$$

Solve the following:

$$e^x = 4$$

$$4^x = 16$$

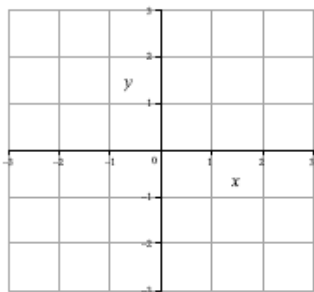
$$9^{x+1} = 3$$

$$\ln(x+1)^2 = 2$$

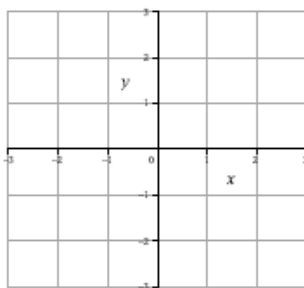
$$2 \ln 3x = 19$$

$$e^{2x} - 4e^x - 5 = 0$$

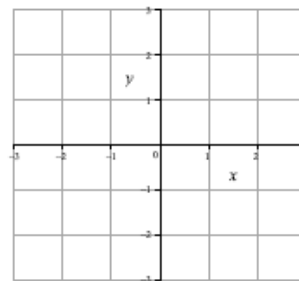
Sketch the graph of each function and state its domain:



$$f(x) = \ln x$$



$$f(x) = e^x$$



$$f(x) = 1 - 2^{-x}$$