Calculus

Summer Packet 2019-2020 Name: Per:______

This packet is to help you review various topics that are considered to be prerequisite knowledge upon entering Calculus. In order to ensure that the good skills that you developed in your PreCalculus course do not disappear this summer, working on this packet is highly recommended over the summer. (A good habit would be do at least one math problem every day.) Enjoy your summer, but be sure to come prepared with the necessary knowledge to continue on into Calculus next year. There will be a skills quiz on these topics in the fall.

I. Algebra

Factor the following

$x^2 - 2x - 8$

$4x^2 + 5x - 6$

$2x^2 + xy - 15y^2$

$x^2 - 4$

$a^4 - b^4$

$x^2 - 4x + 4 - y^2$

Perform the indicated operations and simplify

$$\frac{x^2 - x - 6}{x^2 - 6x + 9}$$

$$\frac{x^2 + 4x + 4}{x^2 - 3x - 4} \cdot \frac{2x^2 + x - 1}{x^2 - 3x - 10}$$

$$\frac{2x^2 - 6x}{2x^2 - x - 10} \div \frac{x^2 - 5x + 6}{x^2 - 4}$$

$$\frac{1}{x - 1} + \frac{3}{x + 2}$$

$$\frac{x}{2x-1} - \frac{3}{x+2}$$

$$\frac{\frac{1}{a^2} - \frac{2}{ab}}{\frac{1}{2ab} - \frac{1}{b^2}}$$

Use any method to solve the equation:

$6x = 3x^2$

$x^2 - 4x - 5 = 0$

$4x^3 - 6x^2 = 0$

$\sqrt{x+4} = 3$

$|2x+3| = 7$

$\frac{1}{2x} + \frac{5x}{8} = 6$

$2x^2 - 8x = -3$

$(t+1)(2t-1) = 1$

$12 - 2s = 3s^2$

Find the equation of the line (in point-slope form) that passes through each set of points:

$(2, 1),\ (14, 6)$ $(-1, 0),\ (6, 2)$ $(1, 6),\ (4, 2)$

Determine whether the following functions are odd, even, or neither:

$x^2 + 2$ $2x^3 - x$ $x^4 - 3x$

Determine the degree of the following polynomials:

$x^2 + 4x - 5$ $2x - 4 + x^3$ $(x - 2)^2$

Determine the domain and range of each function:

$(x - 1)(x + 2)$ $3 - 2x^2$ $\sqrt{2x^2 - 1}$

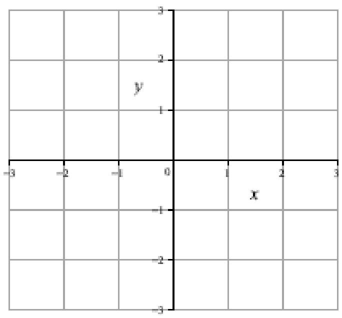
$\frac{2}{3x + 4}$ $|x + 5|$ $\sqrt{36 - x^2}$

$\sqrt{3 - x - 2x^2}$

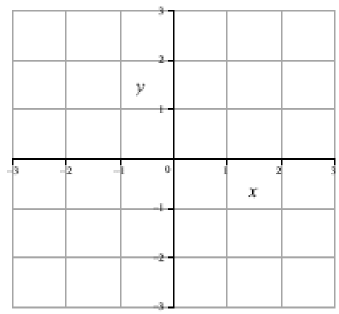
$\sqrt{\frac{x+3}{x^3 - 4x^2}}$

$\frac{1}{\sin x}$

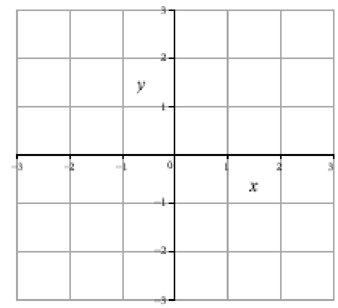
Sketch the graph of each function and determine its domain:



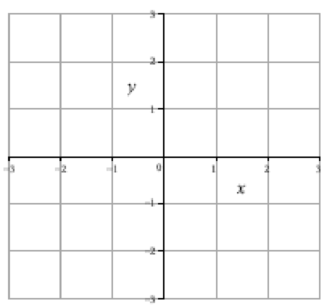
f(x) = $x + 1$



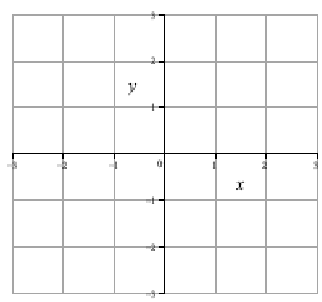
f(x) = $2x - 3$



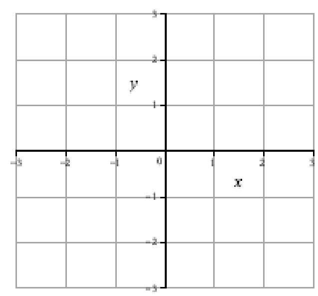
f(x) = $\frac{1}{x-1}$



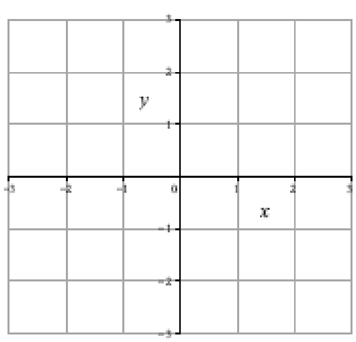
f(x) = $\sqrt{2 - x}$



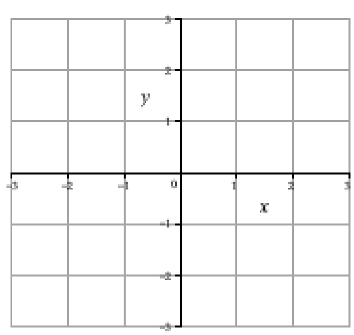
f(x) = $\sqrt{9 - x^2}$



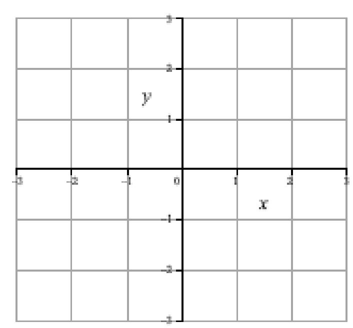
f(x) = $\frac{2}{x^2}$



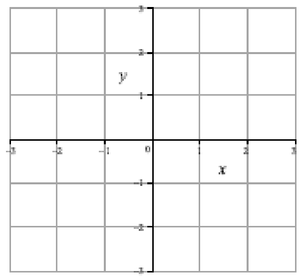
f(x) = $|x - 1|$



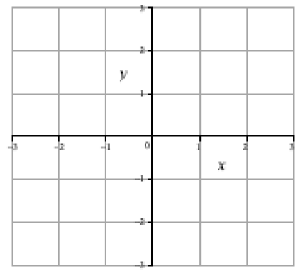
f(x) = $\frac{x^2 - 1}{x + 1}$



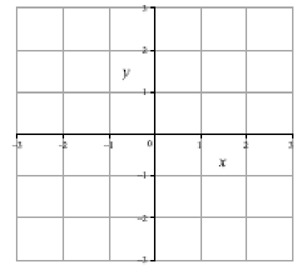
f(x) = $x^2 - 6$



$$f(x) = \begin{cases} 1, & x < 0 \\ x+1, & x \geq 0 \end{cases}$$



$$f(x) = \begin{cases} x^2 - 1, & x < 1 \\ x, & x > 1 \end{cases}$$



$$f(x) = \begin{cases} x+1, & x < -1 \\ 1-x, & -1 \leq x < 1 \\ x, & x \geq 1 \end{cases}$$

Circle which of the following graphs are functions over the domain shown.

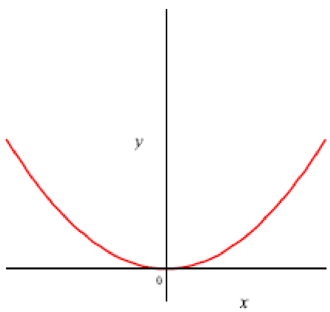


Figure 1

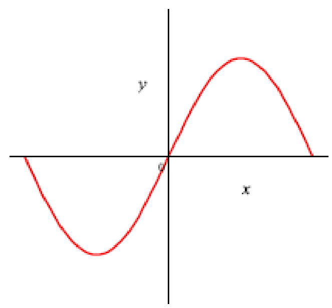


Figure 2

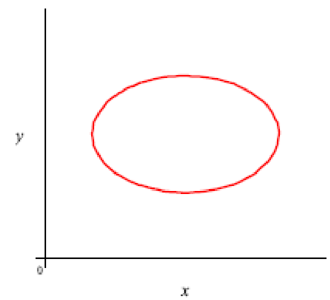


Figure 3

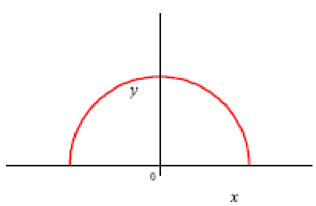


Figure 4

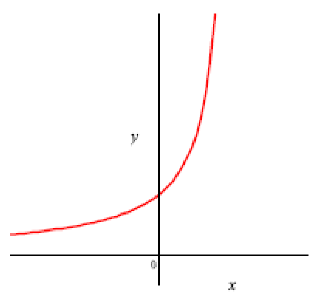


Figure 5

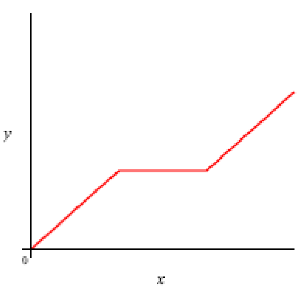


Figure 6

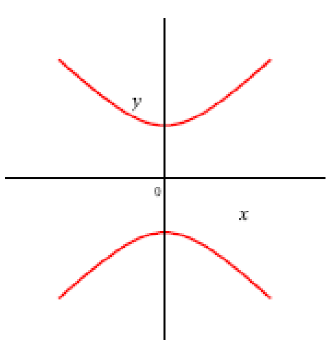


Figure 7

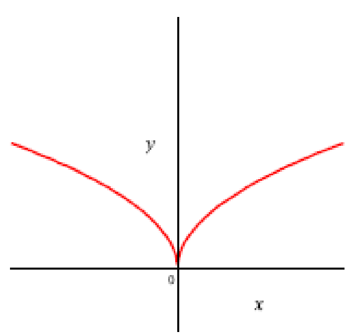


Figure 8

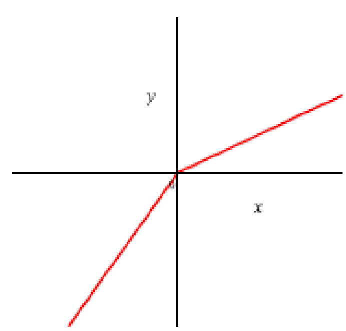


Figure 9

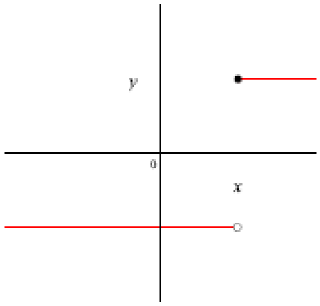


Figure 10

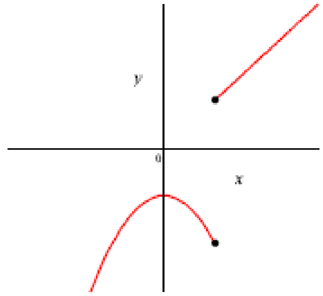


Figure 11

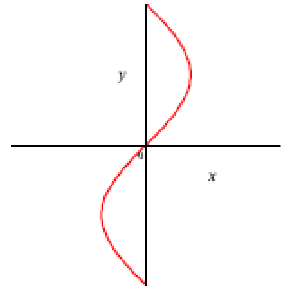
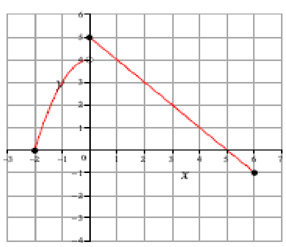


Figure 12

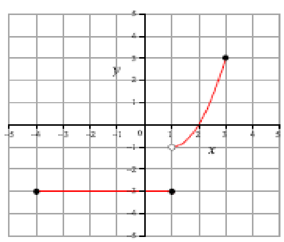
Composition of Functions If $f(x) = x + 2$ and $g(x) = \sqrt{x}$, find each function and its domain:

$(f + g)(x)$ $\quad$ $(f - g)(x)$ $\quad$ $(fg)(x)$ $\quad$ $\left(\frac{f}{g}\right)(x)$

$f(g(x))$ $\quad$ $(f \circ g)(x)$ $\quad$ $g(f(x))$ $\quad$ $f(f(x))$



$y = f(x)$



$y = g(x)$

1) Is 3 in the domain of $g(f(x))$? If it is, find $g(f(3))$. If not, explain why not.

2) Is -3 in the domain of $f(g(x))$? If it is, find $f(g(-3))$. If not, explain why not.

3) Is 4 in the domain of $g(f(x))$? If it is, find $g(f(4))$. If not, explain why not.

4) Find the domain of $f(g(x))$.

Given the function $f(x) = x^3 - 2x^2 + 2x - 4$**, answer the following questions:**

(a) What is the degree of this function?

(b) How many complex roots does it have?

(c) What is the minimum number of real roots that can you be sure that it has?

(d) What is the domain in which all of its real roots must lie?

(e) What are its potential rational roots?

(f) Determine its real root(s) using your calculator

Given the function $f(x) = x^4 + x^3 - 2x^2 - 7x - 4$**, answer the following questions:**

(a) What is the degree of this function?

(b) How many complex roots does it have?

(c) What is the minimum number of real roots that can you be sure that it has?

(d) What is the domain in which all of its real roots must lie?

(e) What are its potential rational roots?

(f) Determine its real root(s) using your calculator

Trigonometry Complete the following tables of useful trigonometric values:

Degrees	0°	30°	45°	60°	90°	180°	270°	360°
Radians								
$\sin\theta$								
$\cos\theta$								
$\tan\theta$								
$\csc\theta$								
$\sec\theta$								
$\cot\theta$								

Degrees	120°		150°		225°		300°		330°
Radians		$\frac{3\pi}{4}$		$\frac{7\pi}{6}$		$\frac{4\pi}{3}$		$\frac{7\pi}{4}$	
Ref. Angle									
Quadrant									
$\sin\theta$									
$\cos\theta$									
$\tan\theta$									

Complete the following:

$\cos^2 x + \sin^2 x =$ __________ $\quad 1 + \tan^2 x =$ __________ $\quad 1 + \cot^2 x =$ __________

$\sin 2x =$ __________ $\quad \tan\left(\frac{\pi}{2} - x\right) =$ __________ $\quad \cos(-x) =$ __________

Find all solutions $0 \le x < 2\pi$ for each of the following.

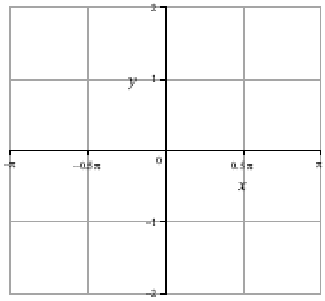
$\tan x - 3\cot x = 0$

$\sqrt{3}\cos x = \sin 2x$

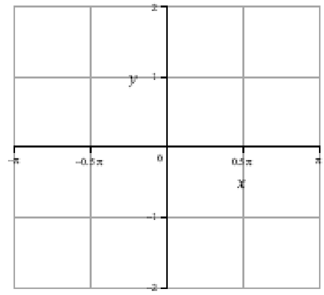
$2\cos^2 x - \cos x - 1 = 0$

$4\cos^2(2x) = 1$

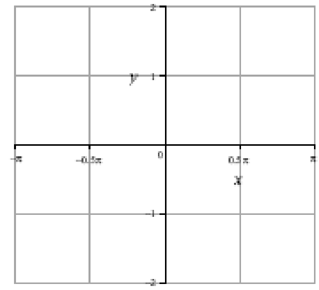
Sketch the graph of each function and state its domain and range:



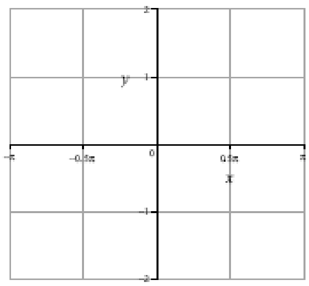
f(x) = $\sin x$



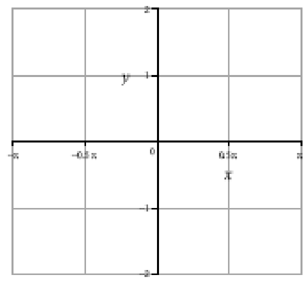
f(x) = $\cos x$



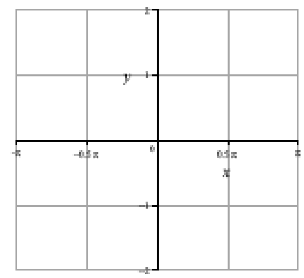
f(x) = $\tan x$



f(x) = $\csc x$



f(x) = $\sec x$



f(x) = $\cot x$

Prove the following:

$$\tan\theta + \cot\theta = \sec\theta\csc\theta$$

$$\frac{2\sin^3 x}{1-\cos x} = 2\sin x(1+\cos x)$$

$$\frac{\csc\theta - \cot\theta}{\sec\theta - 1} = \cot\theta$$

$$\frac{1}{\sec x + 1} = \cot x\csc x - \csc^2 x + 1$$

Logarithms

Use the properties of logarithms and exponents to rewrite each of the following.

2^{x+2} $\qquad$ $\log_6 \frac{5}{x}$ $\qquad$ $\log_{10} x^2y^4$

$2\ln 8 + 5\ln z$ $\qquad$ $\ln \frac{xy}{z}$ $\qquad$ $2\ln 3 - \frac{1}{2}\ln(x^2+1)$

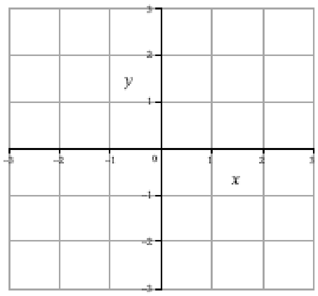
$\log_{10} \frac{x^4\sqrt{x}}{z^4}$ $\qquad$ $\ln e^2$ $\qquad$ $e^{\ln x^2}$

Solve the following:

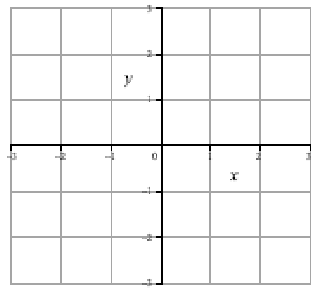
$e^x = 4$ $\qquad$ $4^x = 16$ $\qquad$ $9^{x+1} = 3$

$\ln(x+1)^2 = 2$ $\qquad$ $2\ln 3x = 19$ $\qquad$ $e^{2x} - 4e^x - 5 = 0$

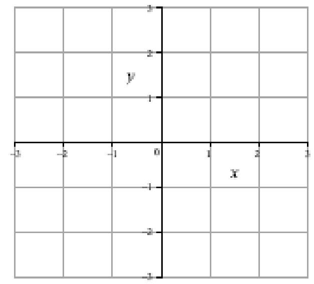
Sketch the graph of each function and state its domain:



f(x) = $\ln x$



f(x) = e^x



f(x) = $1 - 2^{-x}$